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Constituent Quarks, Diquarks and the $N-\Delta$ Mass Splitting *

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We analyze hadron as well as quark and diquark correlation functions in Landau gauge in order to extract information on the spin dependence of the quark-quark interaction. We find evidence that the $N-\Delta$ mass splitting can be attributed to the spin dependence of the interaction between quarks in a colour anti-triplet state with spin 0 and 1, respectively. The lightest excitations are observed in the $S=0$ channel. However, no evidence for a deeply bound diquark state is found.

1. INTRODUCTION

The spin dependent interaction among quarks found in perturbation theory from one gluon exchange diagrams (OGE) [1] as well as induced by instantons leads at least to a qualitative understanding of the fine structure of the hadron spectrum [2]. In particular, the attractive interaction in the $S=0$ channel has led to the idea that diquarks may be well defined, localized objects (bound states?) within a nucleus. This also led to the speculation that diquarks may play an important role for the phase structure of QCD at high density. The possibility of a diquark gas [3] has been discussed and recently new ideas about the possible existence of a diquark condensate (*colour superconductor*) [4–6] have been put forward.

Whether these newly proposed phases of dense matter are realized in nature depends on details of the spin dependent part of the q-q interaction. Also in the case of the hadron spectrum it is of interest to analyze the relative importance of the different mechanisms that can give rise to the observed spin splitting of hadronic states. Already in the case of the $N-\Delta$ mass splitting the contributions from $S=0$ and 1 channels differ in the case of OGE and instanton induced interactions. Both lead to attractive terms for spin 0 diquarks in a colour anti-triplet ($\bar{3}_c$) state they predict, however, opposite signs for the interaction in the $S=1$ channel (see Table 1).

While calculations within the instanton liquid model gave indications for a quite light (deeply bound) diquark state [7] an analysis of the quark distribution inside a nucleus did not give any hints for well localized diquarks [8]. In the following we will present some results from our analysis of the q-q interaction performed in Landau gauge within the quenched approximation of QCD [9].

(F,S,C)	state	Inst.	OGE
$(\bar{3},0,\bar{3})$	$\epsilon_{abc}(C\gamma_5)_{\alpha\beta}u^\dagger_{a,\alpha}d^\dagger_{b,\beta}$	-2	-2
$(6,1,\bar{3})$	$\epsilon_{abc}u^\dagger_{a,\alpha}u^\dagger_{b,\alpha}$	-1/3	2/3
$(\bar{3},1,6)$	$u^\dagger_{c,\alpha}d^\dagger_{c,\alpha}$	2/3	-1/3
$(6,0,6)$	$(C\gamma_5)_{\alpha\beta}u^\dagger_{c,\alpha}u^\dagger_{c,\beta}$	1	1

Table 1
Diquark states with spin S and different flavour (F) and colour (C) representations. The last two columns give the relative strength of interaction terms corresponding to a flavour-spin (instanton) and colour-spin (OGE) coupling, $V_S \sim (\lambda^a_1\lambda^a_2)(s_1 s_2)$, where λ^a_i denote $SU(3)$ generators.

The calculations we are reporting here are based on an analysis of 73 gauge field configurations, generated on $16^3\times 32$ lattices at $\beta=4.1$ with a tree-level Symanzik improved action. After fixing the Landau gauge fermionic correlation functions have been calculated with four different source vectors using a tree-level improved clover action. Propagators have been analyzed for 8 values of the hopping parameter, $\kappa\in[0.14,0.148]$, which corresponds to the hadron mass interval $0.5 < m_\pi/m_\rho < 0.9$. From a calculation of the string tension we find the lattice cut-off, $a^{-1}\simeq 1.1$ GeV; our largest κ-value corresponds to a pion mass $m_\pi=(0.316\pm0.003)a^{-1}\simeq 350$ MeV.

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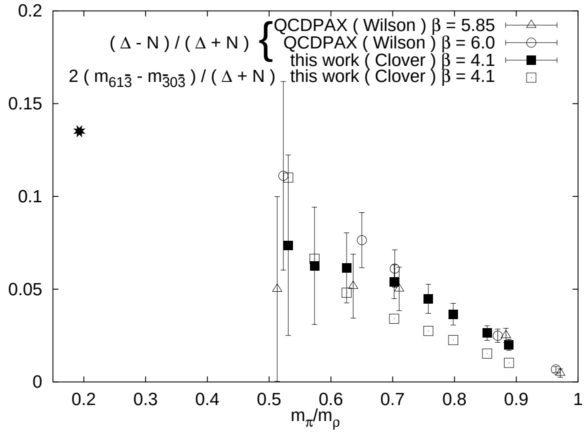

Figure 1. The $N-\Delta$ mass splitting versus the pion-rho mass ratio. The star denotes the experimental value. Data for Wilson fermions are taken from [10]. Also shown is the mass splitting between $S=0$ and 1 diquarks (open squares). Errors are similar to those shown for the $N-\Delta$ mass splitting.

2. $N-\Delta$ MASS SPLITTING

On our data set we have calculated correlation functions for the nucleon and delta. We find that the $N-\Delta$ mass difference increases with decreasing quark massi. As shown in Fig. 1 results are also quantitatively consistent with earlier findings from calculations with Wilson fermions [10].

Simple constituent quark models relate the $N-\Delta$ mass splitting to the spin dependence of the interaction among quarks in a $\bar{3}_c$ state, *i.e.* $m_\Delta - m_N = 2(V_{S=1}-V_{S=0})$. In order to test in how far such a relation can hold we have calculated diquark correlation functions in Landau gauge,

$$G_{F,S,C}(t) = \langle D(0)D^\dagger(t)\rangle \quad . \tag{1}$$

Here $D(t)$ denotes one of the states given in the second column of Table 1. At large distances the correlation functions are expected to decay exponentially with a mass which is characteristic for the given quantum number channel. In Fig. 2 we show the ratio of correlation functions for $\bar{3}_c$ diquarks with $S=0$ and 1,

$$G_{\bar{3}0\bar{3}}(t)\ /\ G_{61\bar{3}}(t) \sim \exp\bigl([m_{61\bar{3}} - m_{\bar{3}0\bar{3}}]\ t\bigr) \quad . \tag{2}$$

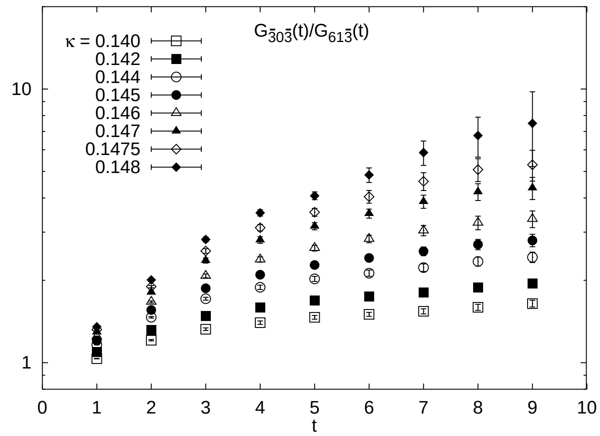

Figure 2. Ratio of correlation functions of colour anti-triplet states with spin $S=0$ and $S=1$ versus Euclidean time t.

The ratio of correlation function clearly rises exponentially at large distances; the slope increases with decreasing quark mass. The mass differences extracted from fits to this correlation functions give, again within simple potential models, the difference between the spin-dependent q-q interactions, $m_{61\bar{3}} - m_{\bar{3}0\bar{3}} \equiv (V_{S=1}-V_{S=0})$. Twice this difference is shown in Fig. 1 (open squares). As can be seen it is quite compatible, although systematically below the results for the $N-\Delta$ mass splitting.

3. OGE VERSUS INSTANTONS

In order to differentiate between predictions of OGE and instanton induced interaction models for different quantum number channels one may compare the $S=1$ correlators in $\bar{3}_c$ and 6_c channels. The ratio $G_{\bar{3}16}(t)/G_{61\bar{3}}(t)$ is expected to rise (fall) with increasing t in the case of OGE (instanton) induced q-q interactions (Table 1). From the results shown in Fig. 3 it is obvious that at least for heavy quarks the interaction in the $\bar{3}_c$ channel leads to lighter excitations, *i.e.* is more attractive. This is in favour of the instanton induced interaction models. We note, however, that with decreasing quark mass there is a tendency for the ratios to flatten. A t-independent ratio in the chiral limit thus cannot be ruled out. This could

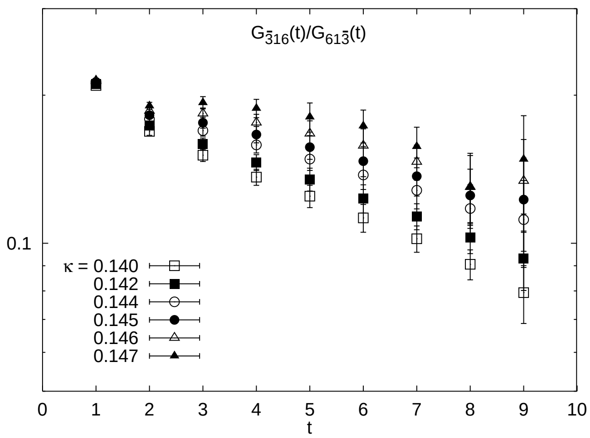

Figure 3. Ratio of correlation functions of $S = 1$, colour $\bar{3}$ and 6 states versus Euclidean time t.

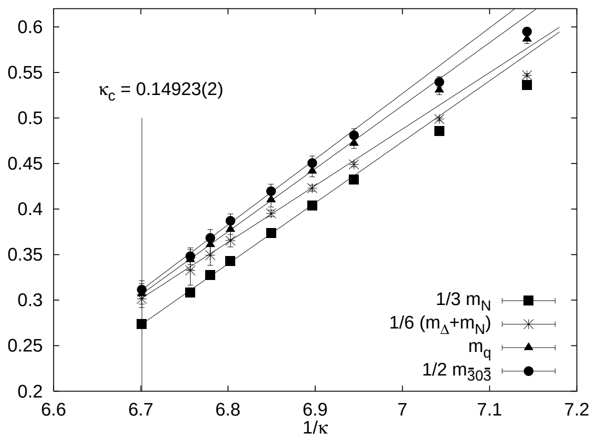

Figure 4. Masses in units of the number of valence quarks for different values of κ.

suggest that no bound states exist in these quantum number channels.

4. QUARK AND DIQUARK MASSES

The above discussed analysis suggests that the diquark correlator in the $\bar{3}_c$, spin zero channel leads to the lightest excitations. The correlation function $G_{\bar{3}0\bar{3}}(t)$ does indeed show a rather clean exponential decay, which leads to a plateau for local masses at distances $t \sim 4$ for all quark masses considered by us. This suggests that the diquark is a well localized state. Using the string tension to set the scale ($\sqrt{\sigma} = 420$MeV) we find from an extrapolation to the chiral limit $m_{\bar{3}0\bar{3}} = 694(22)$MeV.

A similarly stable behaviour is found for local masses extracted from the quark propagator. Unlike for all diquark masses the plateau is, however, reached from below which reflects the non-existence of a positive transfer matrix in Landau gauge. In the chiral limit we find for the constituent mass, $m_q = 342(13)$MeV. The mass of the lightest diquark obviously is consistent with twice the constituent quark mass. There thus is no evidence for a deeply bound diquark state as it has been found in instanton model calculations. A comparison of quark, diquark and hadron masses is given in Fig. 4. As can be seen 1/3 of the nucleon mass lies below the quark mass which can be addressed to the contribution of an attractive q-q interaction.

The analysis of diquark correlation functions in different quantum number channels shows that correlations are stronger in colour anti-triplet channels than in colour sextet channels. Nonetheless there is so far no evidence for a deeply bound diquark state. One should, however, stress that this first exploratory lattice study has been performed in the quenched approximation on quite coarse lattices and still with fairly large quark masses. The influence of these approximations should be analyzed in more detail in the future.

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